QUESTIONS AND ANSWERS

Contributions to this section, both Questions and Answers, are welcomed. Please submit four copies to the editorial office. Please include a *title* for each submission, include name and address at the end, and put references in the standard format used in the American Journal of Physics. For further suggestions, sample Questions and Answers, and requested form for both Questions and Answers, see Robert H. Romer, "Editorial: 'Questions and Answers,' a new section of the American Journal of Physics," Am. J. Phys. **62** (6) 487–489 (1994).

Questions at any level and on any appropriate AJP topic, including the "quick and curious" question, are encouraged.

Question #55. Are there pictorial examples that distinguish covariant and contravariant vectors?

The distinction between covariant and contravariant vectors is necessary when dealing with curved spaces. I am interested in collecting *pictures* that one can draw on a chalkboard or on paper that *show visually* in some nontrivial examples the distinction between covariant and contravariant vectors (or, more generally, between a vector space and its dual).

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Question #56. Ice cream making

Everyone who makes ice cream, at least everyone who makes it in the correct way (turning the crank by hand), knows that the freezing point of water with a heavy dose of salt is considerably less than 0 °C. Likewise, the boiling point of such a solution is elevated above 100 °C. I would like to have a simple explanation of one or both of these familiar facts, either in terms of familiar facts about entropy or in terms of the molecular structure of water. Are the two facts, lowering of the freezing point and elevation of the boiling point, related to one another? I also wonder whether the suppression of the freezing point is related in any way to the fact that water expands upon freezing.

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Answer to Question #45 ["What (if anything) does the photoelectric effect teach us?," R. Q. Stanley, Am. J. Phys. 64 (7), 839 (1996)]

Stanley[1] points out that "the familiar facts of the photoelectric effect" are explainable without photons. He raises questions as to (1) literature clarifying the situation for people not well versed in quantum field theory; (2) what should be said to students "without being overwhelming;" and (3) a discussion of more subtle aspects of the photoelectric effect that do require field quantization.

I too find it curious that so many textbooks point to the photoelectric effect as proof for the existence of photons. A treatment using classical electromagnetism was given by Wentzel[2] in 1927—the same year that Schrödinger,[3] incidentally, explained the Compton effect without field quantization! That field quantization is not required for an explanation of the photoelectric effect has been known for many years by optical physicists. A succinct and entertaining account is given by Franken,[4] whose paper I suggest as an answer to Stanley's question (1) above. His discussion assumes familiarity with Fermi's golden rule, but I can think of no simpler way to approach the subject in the "honest and correct" way sought by Stanley. Various authors[5] have reviewed the history and misunderstanding of the photon concept, and Lamb,[5] in particular, has cautioned repeatedly against naive interpretations of photons.

What should be said to students? The following circumstances might be noted. Einstein's 1905 paper[6] was concerned mainly with blackbody theory; by comparing the change in entropy of an ideal gas undergoing isothermal expansion to the corresponding change for radiation *satisfying the Wien limit* ($h\nu \gg kT$), Einstein concluded that such radiation acts as if it consists of independent particles of energy $h\nu$. He then applied this concept to the photoelectric effect and predicted the linear relation between radiation frequency and stopping potential that would later be validated by Millikan.[7] Einstein's work preceded any quantum theory of electrons—Planck's constant appeared via the formula $E=h\nu$ for light quanta (photons). Later it became possible to obtain Einstein's relation by treating the electrons quantum mechanically and the field classically. In this approach[2–5] h enters through its appearance in the Schrödinger equation for the electrons. It allows Einstein's relation to be deduced without photons: Once electrons are described by the Schrödinger equation, it follows that a *classical light wave* of frequency ν can induce an electron to change its state if the change in electron energy is $h\nu$.

There are, to be sure, effects that require photons for their explanation, but the familiar facts of the photoelectric effect are not among them. One easily understood "proof" of the reality of photons is the fact that an atom recoils when it radiates: If the emitted radiation is simply a classical wave, no radiation pattern having the atom as the center of inversion symmetry (e.g., a spherical wave) can carry linear momentum, and consequently the atom cannot recoil. On the other hand, if the radiation is emitted in some one direction as a photon, then the atom must recoil. Such recoil is observable in atomic beam experiments,[8] and it plays an important role in recent developments in the cooling and trapping of atoms with lasers. It was predicted by Einstein,[9] who concluded that "Outgoing radiation in the form of spherical waves does not exist," based on the presumed consistency of the Planck spectrum with the Maxwell–Boltzmann distribution for the motion of the atoms.

If the radiation from a single atom were a classical wave spread over some region of space, it would be possible for the spontaneous emission of a single atom to cause the excitation of *two* atoms, in apparent violation of energy conservation. Similarly, if a beam of light is divided by a beam splitter into two beams, and each of these beams is incident on a photodetector, it would be possible according to classical wave theory to count photoelectrons at each detector even if the original beam is so dim that its energy corresponds to one photon. An experiment by Clauser shows in effect that this does not occur, and thereby ''resurrects the photoelectric effect as a phenomenon requiring quantization of the electromagnetic field.''[10] In other words, there are indeed ''some subtle features of the photoelectric effect that *do* demand quantization of the field.''[1]

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[1] R. Q. Stanley, Am. J. Phys. **64**, 839 (1996), Question #45.

[2] G. Wentzel, ''Über die Richtungsverteilung der Photoelektronen,'' Z. f. Phys. **41**, 828–832 (1927).

[3] E. Schrödinger, ''Über den Comptoneffekt,'' Ann. d. Physik **82**, 257–264 (1927). An English translation may be found in Schrödinger's *Collected Papers on Wave Mechanics* (Blackie & Sons, London, 1929), pp. 124–129.

[4] P. A. Franken, ''Collisions of Light with Atoms,'' in *Atomic Physics*, edited by B. Bederson *et al.* (Plenum, New York), pp. 377–387 (1969).

[5] See, for instance, M. Scully and M. Sargent III, ''The Concept of the Photon,'' Physics Today, 38–47 (March 1972); P. W. Milonni, ''Wave-Particle Duality of Light: A Current Perspective,'' in *Wave-Particle Dualism, Papers in Honor of the 90th Birthday of Louis de Broglie*, edited by S. Diner, D. Fargue, G. Lochak, and F. Selleri (Reidel, Dordrecht, 1983), pp. 27–67; W. E. Lamb, Jr., ''Anti-Photon,'' Appl. Phys. B **60**, 77–84 (1995).

[6] For an English translation of Einstein's paper see, for instance, A. B. Arons and M. B. Peppard, Am. J. Phys. **33**, 367–374 (1965).

[7] R. A. Millikan, ''Einstein's Photoelectric Equation and Contact Electromotive Force,'' Phys. Rev. **7**, 18–32 (1916); ''A Direct Photoelectric Determination of Planck's 'h,' '' Phys. Rev. **7**, 355–388 (1916).

[8] See J. L. Picque and J. L. Vialle, ''Atomic-Beam Deflection and Broadening by Recoils Due to Photon Absorption or Emission,'' Opt. Commun. **5**, 402–406 (1972).

[9] A. Einstein, ''Zur Quantentheorie der Strahlung,'' Phys. Zs. **18**, 121–128 (1917).

[10] J. F. Clauser, ''Experimental distinction between the quantum and classical field-theoretic predictions for the photoelectric effect,'' Phys. Rev. D **9**, 853–860 (1974). The experiment involved checking for ''extra'' photoelectron coincidence counts predicted by classical wave theory. A recent experiment of J. D. Franson [Phys. Rev. A **53**, 3756–3760 (1996)] indicates that there is a very small probability of splitting a single photon into two photons. However, this has to do with a quantum-electrodynamical nonlinear interaction that is *energy-conserving* and therefore does not contradict Clauser's conclusions.

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Answer to Question #46 [''How does the microwave oven really work?,'' Clifford Swartz, Am. J. Phys. 64(7), 839 (1996)]

The notion that microwave ovens exploit a ''resonance'' of the water molecule is, as Clifford Swartz recognizes, a piece of nonsense, demonstrating the power of the mystical term ''resonance'' to paralyze thought. Many years ago on an examination I asked students to choose the optimum frequency at which microwave ovens should operate. To help them make their choice, I included a plot of the complex dielectric function of water at frequencies (inverse wavelengths) between 0.01 and 10 cm^{-1}. Careless students picked the frequency at which absorption (strictly, the imaginary part of the dielectric function) peaked. Thoughtful students reasoned as follows. Cooking in a microwave oven entails heating the water in food by absorption of microwave radiation. A parallel beam of (monochromatic) radiation is attenuated with distance x in a homogeneous medium according to

$$I=I_0e^{-\alpha x}, \tag{1}$$

where α is the absorption coefficient of the medium (which follows from its complex dielectric function), I is the power density (power per unit area), and I_0 is the power density at $x=0$. The rate of energy deposition (per unit volume) in the medium is the spatial derivative of this power density (sometimes called the flux divergence):

$$\alpha I_0e^{-\alpha x}. \tag{2}$$

Choosing the frequency at which α is greatest (assuming that this will yield $\alpha x \gg 1$) is clearly not the best strategy. This will result in a high heating rate confined to the surface of the object heated. To ensure a more or less uniform heating rate, choose the frequency for which $\alpha x \approx 1$. Take x to be a characteristic length of the largest object to be heated, a pot roast, say, which is mostly water. Thus the optimal frequency is that for which α for water is $1/x$, where x is the thickness of a roast.

When I gave this problem, I did not know the frequency at which microwave ovens operate. But when the students (at least some of them) and I did the previous crude analysis, we obtained a frequency remarkably close to the correct value, which, by the way, is considerably lower than the frequency of peak microwave absorption by water.

I don't agree with Swartz that the response of liquid water ''cannot be trivially calculated,'' setting aside what exactly is meant by ''trivially.'' A simple theory of the dielectric response of liquid water is given by Debye in his classic treatise on polar molecules.[1] At microwave frequencies, liquid water molecules, with their permanent electric dipole moments, are essentially overdamped oscillators. The mode of electric polarization of these molecules is referred to as *relaxation*. Debye obtained a good estimate for the relaxation time of water molecules by treating them as spheres in a viscous fluid (water). Debye's simple theory also yields the temperature dependence of the relaxation time. At microwave frequencies liquid water does not have a resonant frequency, but even if we were to define its resonant frequency as the frequency of peak absorption, this still would not be the frequency at which microwave ovens operate.

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[1] P. Debye, *Polar Molecules* (Chemical Catalog Co., New York, 1929), Chap. 5.

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